

Modeling the Failure and Flow of Anisotropic Cracked Sea Ice

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LONG TERM GOALS

My ultimate goal in this work is to develop a capability and physical basis to simulate oriented fractures and leads in sea ice in a way that can be utilized in the next generation of sea ice models.

OBJECTIVES

The main objective of this effort is to develop and numerically investigate a physically based model for describing and simulating the failure and flow of anisotropic cracked sea ice, and ultimately simulate oriented fractures on the geophysical scale in sea ice. With this objective in mind a conceptual model based on oriented lead ice imbedded in thicker ice has been formulated numerically with the yield criterion for both the thick and thin ice taken from laboratory observations together with an associated non normal flow rule prescribed in an energetically consistent manner. I then plan to utilize this model together with wind and water stress forcing on the ice pack to simulate oriented fractures in pack ice and compare these features with observations from satellite imagery. Secondary related objectives are (1) to develop a capability to simulate high frequency variability in pack ice as this should prove critical in the *actual formation of opening leads* as opposed to sliding fractures and (2) develop an energetically consistent ice dynamics model in curvilinear co-ordinates to be used both in forecast applications and for consistently determining positive definite mechanical energy dissipation for use in ambient noise acoustic models for the Arctic.

APPROACH

A conceptual notion especially relevant to the geophysical scale is that leads and flaws develop from the interaction and propagation of existing oriented flaws. To formulate this numerically, the fundamental model consists of an oriented weak lead/flaw imbedded in thicker ice as shown in Figure 1.

Commensurate with continuum mechanics postulates the interfacial stresses (σ_{11} and σ_{12}) along the lead are taken to be the same in the thick and thin ice while stresses perpendicular to the lead strike (σ_{22}) may be different. For the basic numerical studies the isotropic constitutive laws for both the thick and thin ice are taken to be coulombic in character (Figure 1-b) with slopes and tensile stresses consistent with laboratory observations. Other constitutive laws are however also utilized. An associated non normal flow rule for this yield curve is constructed to insure energetic consistency (i.e., the quantity $\Sigma_{ij} \dot{\epsilon}_{ij} \sigma_{ij}$ is always positive definite). In addition the transition from coulombic flow to out of plane failure is taken to occur at the transition between diverging and converging flow. For numerical investigation this model is generalized to include oriented leads in a variety of directions. A series of numerical investigations of this model under both near field stress forcing and far field stress are carried out. In the far field stress forcing experiments oriented flaws may interact with other

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oriented flaws at different locations. These results are analyzed to determine basic fracture propagation characteristics as well as deformation behavior with the goal of simulating oriented fractures and leads.

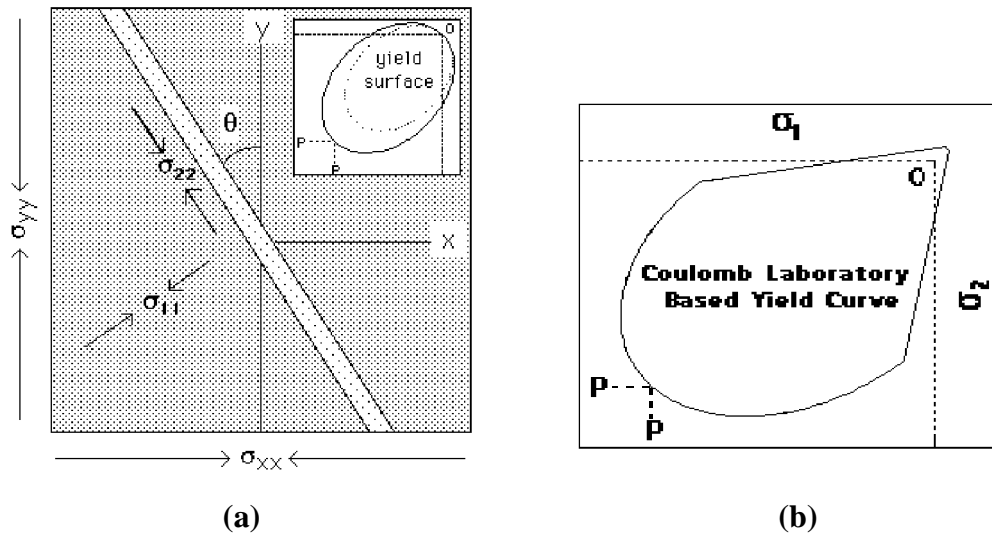


Figure 1. (a) Schematic of oriented thin ice lead or flaw embedded in thick ice. An isotropic elliptical yield curve for both the thin and thick ice is shown in the inset. (b) Laboratory based coulombic yield curve. Stresses above or to the right of the dotted lines are tensile.

WORK COMPLETED

A series of numerical experiments using several basic yield curves were carried out in order to extensively numerically investigate the near and far field failure characteristics of the basic model. Numerical experiments were also carried out to determine precisely where the transition between opening and closing leads occurs and the necessary conditions for energetic consistency. Both analytic and numerical investigations were carried out to elucidate the physical principles underlying the modification of the aggregate coulombic failure for this model even though there is a constant slope failure in the basic yield curve. To understand the far field behavior a series of far field experiments were carried out with both interacting and isolated oriented flaws surrounded by thicker ice (Hibler and Schulson, 1998). To develop a true fracture propagation model a general procedure for integrating the yield curve was initiated.

With respect to other work, a procedure for imbedding the ice in the boundary layer was developed for investigating the interaction between propagating kinematic waves and inertial oscillations. This model (Hibler et. al., 1998) was used in conjunction with observational data on ice deformation to determine if this mechanism could be responsible for much of the high frequency variability that is a ubiquitous feature of observed ice deformation.

Finally, a differencing procedure for curvilinear co-ordinates was developed that insures that (a) the globally integrated work done by ice interaction forces equals the global mechanical energy dissipation, and (b) the mechanical energy dissipation is always positive definite. For momentum conservation it was found that this differencing was considerably different than that used in rectangular co-ordinates.

RESULTS

The essential result is that with an anisotropic model of a single flaw preferred lead orientation occurs under near field stresses for ice compressed in a biaxial state. In contrast to traditional coulombic conjugate plane theory of faults for which a constant orientation would be expected, the preferred lead orientation depends on the confinement ratio with lower confinement ratios producing leads more closely aligned with the largest principal stress with the intersection angle going to zero for large enough positive confinements. If this result is formulated in terms of an effective friction coefficient the results show a *decrease in the effective friction for larger confinement*, a result consistent with observations in rock mechanics. Analysis of the results indicates that these features arise because of the finite width of the lead allowing compressive stress along the lead.

While for a given confinement ratio the precise orientation of the leads does depend on the constitutive law assumed for the thin and thick ice, if we consider a flaw imbedded in surrounding stronger ice and apply a far field stress to that region the behavior is quite different. In this instance we find that the local stress field is modified by the presence of the oriented flaw which in turn modifies the nearby maximum tensile stresses (Figure 2). In the case of different flaw orientations we find that there is a much narrower range of orientations (relative to the far field stress) that yield the greatest tensile stresses in the neighborhood of the flaw. In a manner relatively independent of strength, flaws oriented at about 20° relative to the far field stresses tend to yield the most pronounced tensile stresses. As shown in Figure 2 this characteristic is somewhat, but not significantly, affected by the isotropic rheology assumed for the thin and thick ice. These results and theoretical model are described in a journal article by Hibler and Schulson (1998).

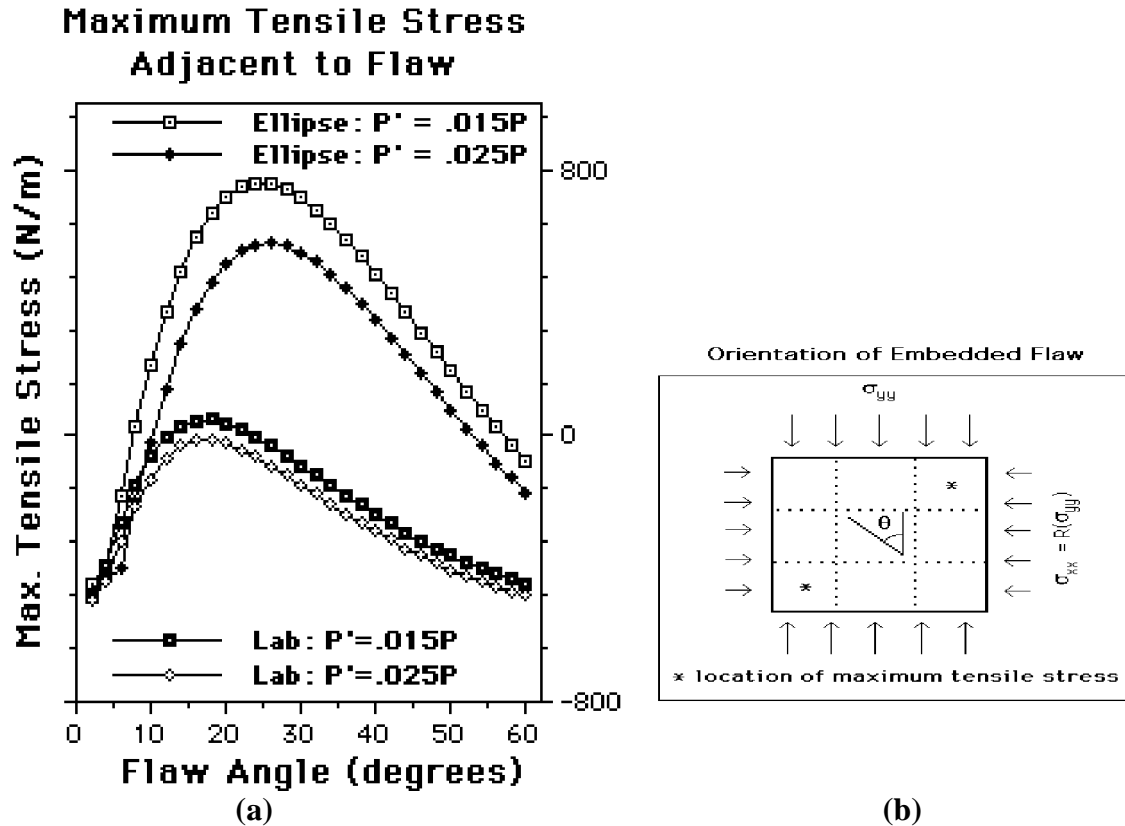


Figure 2. (a) Maximum tensile stress adjacent to flaw as a function of angle of the flaw relative to the far field forcing. Curves for different strengths of flaws are shown with an elliptical yield curve as shown in the inset of Figure 1a. The laboratory yield curve is a mohr coulombic type yield curve as shown in Figure 1b. (see Hibler and Schulson, 1998). (b) Orientation of embedded flaw surrounded by isotropic ice under far field forcing.

These far field results suggests a different mechanism than conjugate coulomb faulting from a parent isotropic coulomb rheology is responsible for lead formation. With this far field result in mind we can, for example, imagine that a complex stress field existing in pack ice (due predominately to wind and water stresses) will tend to favor the weakening and further propagation of flaws with orientations roughly at 20° relative to the 'far field' pack ice stress. Other leads will tend to close and strengthen or weaken more slowly. Because of this selection phenomenon a good first approximation to this phenomenon may be gotten by beginning with a uniform array of oriented flaws in every grid cell. The complex stress field will eventually lead to the failure of two intersecting leads at some point, only one of which will survive a loading of the stress field because of the unstable character of the two flaw system. Once any oriented fracture forms in this manner the system then becomes locally anisotropic. As the stress state increases or the formed flaw weakens by opening, the nearby tensile stresses will activate other adjacent pairs of oriented leads. As before, only one of these leads will survive and the fracturing process will continue with the failure not only occurring over a narrow region but also containing oriented flaws.

To examine high frequency variability issues, an idealized 1.5 dimensional dynamic-thermodynamic sea ice model was configured to be relevant to results of a winter deformation experiment in a tide free region of the East Antarctic sea ice zone (Worby and others, 1996). During this time, a number of drifting buoys were monitored hourly for studies of sea ice drift and deformation (Heil and others, 1998) with results yielding substantial inertial signals in the deformation. The model configuration consisted of a 40km lagrangian grid with wind forcing taken to be perpendicular to the coast. In this model (Hibler et. al., 1998), full two dimensional ice dynamics is assumed but all variations along the coast are assumed to be zero. For simplicity pressure gradient variations in the ocean due to Ekman convergence were also neglected.

Simulations with this grid with different wind forcing show that without an improved boundary layer formulation both the ice drift and deformation follow the wind forcing yielding no inertial deformation. With an improved boundary layer formulation but no ice thickness evolution equations ice drift does show an inertial signal, but there is no inertial deformation signal, except at one grid cell right at the coast. With thickness strength coupling, however, there is a substantial signal in the ice deformation with kinematic waves (see e.g., Hibler et. al., 1983) propagating outward interacting with the inertial motions to produce varying deformation in regions far from the coast.

These oscillations are notably present even in the absence of wind as shown in Figure 3. In this figure we show deformation results after a spatially constant pulse wind was applied to the system for about 4 hours and then dropped to zero. As can be seen there is a substantial change in the ice concentration with eventually a "banded" structure developing with oscillating regions of thin ice alternating spatially with ice undergoing no deformation. Because of the non-linear sea ice dynamics there is only a weak damping of these oscillations although they will eventually die out leaving "banded" ice concentrations behind. Comparison of the magnitude of these oscillations with those observed during quiescent wind conditions (Heil et. al., 1998) show them to be very close in magnitude.

ICE CONCENTRATION VS TIME AND DISTANCE FROM SHORE

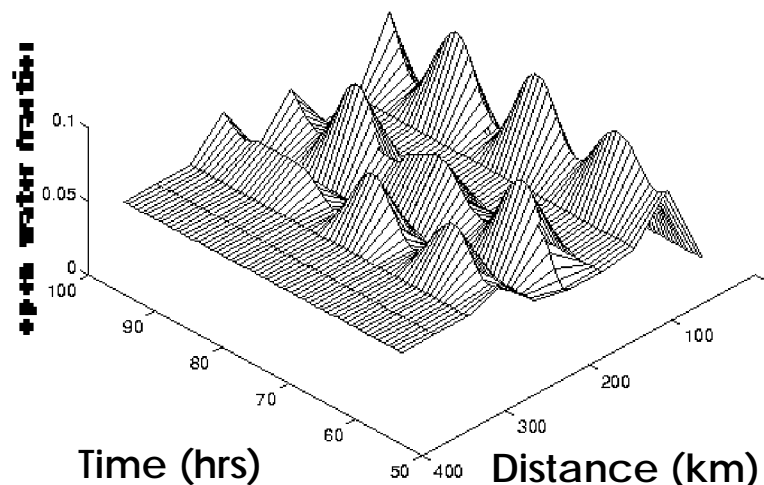


Figure 3. Perspective view of compactness as a function of time and distance from shore about fifty hours after an initial pulse of constant onshore wind stress.

Finally with regard to mechanical energy dissipation in sea ice dynamics models, an energetically consistent finite difference formulation in spherical co-ordinates was developed. Tests with the cartesian co-ordinate version of this model (Hibler and Ip, 1995) showed that the addition of energetic consistency led to a substantial improvement in the agreement between simulated and observed buoy drift, especially as regards excessive stoppage of ice motion. Formulation of the energetic consistent equations in curvilinear co-ordinates was found to require that all metric terms be incorporated into the differencing scheme. Analytical proofs of the differencing scheme were constructed which showed explicitly both momentum and energy conservation. Numerical results together with documentation of the model and development of the continuum equations are included in a report (Hibler, 1998) prepared under the auspices of the Arctic Chair and the Naval Postgraduate School for inclusion in ongoing PIPS modeling efforts.

IMPACT/APPLICATIONS

These results are beginning to have an impact on ice forecasting especially within the context of mechanical energy output for use in acoustic applications. The theoretical development of anisotropic models are also impacting both laboratory and field observations and thoughts on the basic mechanisms of lead and fracture formation.

TRANSITIONS

The main transition is the development and documentation of a fully energy conserving curvilinear co-ordinate model for use in PIPS 3.0 and in ambient noise acoustic applications.

RELATED PROJECTS

Closely related to this project is one by Erland Schulson under a separate ONR grant N00014-97-1-0211 on laboratory experiments on both the small and intermediate scale to observe mechanisms and characteristics of ice fracturing and failure. The synergy between these efforts on the mechanisms for fracture have substantially aided in the modeling work and selection of numerical experiments.

REFERENCES

- Heil, P., V. I. Lytle and I. Allison, 1998: Enhanced thermodynamic ice growth by sea ice deformation, *Ann. Glaciol.*, 27, in press.
- Hibler, W. D. III, and E. M. Schulson, 1998: On a Model for the Anisotropic Failure of Flawed Sea Ice, *Journal of Geophysical Research*, accepted for publication.
- Hibler, W. D. III, P. Heil and V. Lytle, 1998: Modeling High Frequency Variability in Antarctic Sea Ice Models, *Annals of Glaciology*, 27, in press.
- Hibler, W. D., III, I. Udin, and A. Ullerstig (1983), On forecasting mesoscale ice dynamics and buildup, *Annals of Glaciology*, 4, 110-115.
- Hibler, W. D. III, and C. F. Ip, 1995: The effect of sea ice rheology on Arctic buoy drift, in *Ice Mechanics*, ASME AMD-Vol. 207, Dempsey and Rajapakse, Eds., Book No. H00954-1995, 255-263.

Hibler, W. D. III, 1998: A curvilinear co-ordinate energy conserving sea-ice dynamics model for PIPS, NPS report, in preparation.

Worby, A. P., N. L. Bindoff, V. I. Lytle, I. Allison and R. A. Massom, 1996: Winter ocean/sea ice interactions studies in the East Antarctic, *EOS Transactions* 77(46), 456-457.

PUBLICATIONS

Hibler, W. D. III and E. M. Schulson, 1997: On modeling sea-ice fracture and flow in numerical investigations of climate, *Annals of Glaciology*, 25, 26-32, 1997.

Hibler, W. D. III, and E. M. Schulson, 1998: On a Model for the Anisotropic Failure of Flawed Sea Ice, *Journal of Geophysical Research*, accepted for publication.

Hibler, W. D. III, P. Heil and V. Lytle, 1998: Modeling High Frequency Variability in Antarctic Sea Ice Models, *Annals of Glaciology*, 27, in press.